Fibration-defined Integer and Non-integer Axes

Mikhail I. Samoilovitch, A.L. Talis, *Central Scientific Research Institute of Technology "Technomash", Moscow, Russia.* E-mail: samoylovith@technomash.ru

Axis of rotation on the angle $2\pi d/p$ is determined by partition of p vertices of the regular polygon on the p subsets with d+1 vertices for each. When $1<d<p/2$ then polygon is a star-polygon. The sides of the star-polygon intersect in certain extraneous points, which are not included among the vertices. Crystallographical and quasicrystallographical axes (d=1) are determined by cyclic subgroups (of order p=2,3,4,5,6,7...) of the Weyl groups of the root lattices of simple Lie groups. The root lattice E_8 incorporates (or they can by embedded in it) all types of root lattices and gives possibility for building of the special algebraic construction – fibre space. L points of fibre space correspondence b points of base and N points of fibre.

By Hopf fibration for E_8 the N points of the fibre can be split into m sets with 1 or 2 special points for each set; therefore:

$L=b\cdot N=m\cdot b(k+c)$, (1)

where $N=m(k+c)$, c=0, 1, 2. The set of b(k+c) points can be split into subsets from k points which define k vectors. Thus, fibration (1) defines an axis of the order:

$p/d=(1/\gamma)\cdot(b(k+c)/k)=(1/\gamma)\cdot(L/N-mc)$ (2)

where $\gamma=1$ or 2 if a cover for fibre group is one or two-sheeted. The choice of L, N, m and c practically defines one of the subsystems of the E_8 root vectors [1].

Equation (2) defines crystallographical and quasicrystallographical (d=1) and non-integer (d>1) axes in the framework of the unique algebraic construction. The non-integer axes are realized in some rod substructures; for instance the collagen structure has 10/3 axis – rotation on the 108°. All non-integer axes, which are determined Hopf fibration for E_8 lattice are considered.

[1] Samoilovich M.I., Talis A.L., *Nanostructures and photon crystals*, "Tehnomash" Publisher, Moscow, 2004, 5-114.

Keywords: fibration, non-integer axes, algebraic construction