The 4d structure of the title compound consists of atoms with sawtooth-like modulation functions and step-like occupation functions (occupation domains, OD). Most of the structural parameters defining OD are concerned with $\gamma =0.36693$, the $c^*$ component of the modulation wavevector. The 3d structure is closely related to the $\beta$-Bi$_2$Ti$_4$O$_{11}$ structure, where the linkage of TiO$_6$ coordination octahedra constructs the host framework providing the one-dimensional tunnel-like space for the accommodation of Bi ions. Domain boundaries are introduced by a kind of the crystallographic shear (CS) operation in the present structure. Namely, the layer unit consisting of Bi$_2$Ti$_2$O$_8^{2-}$ is removed from the $\beta$-Bi$_2$Ti$_4$O$_{11}$ structure, and remained blocks are displaced to fill the gap. The negative charge of the removed unit is compensated by the substitution of Ba$^{2+}$ ions for Bi$^{3+}$ ions in tunnels.

The unique character of this incommensurate structure is the aperiodic insertion of domain boundaries in contrast to usual (i.e. commensurate) CS structures. To the best of our knowledge, the present study is the first example of the quantitative analysis of the incommensurate CS structure.

**Keywords:** incommensurate structures, higher-dimensional structure analysis, crystallographic shear structures