Knotted Nets and Weavings – from 2D Hyperbolic to 3D Euclidean Patterns

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Crystalline patterns in 3D euclidean space can be constructed from tilings of 2D hyperbolic space, followed by projection onto three-periodic hyperbolic surfaces. We demonstrate the technique by projection onto the simpler three-periodic minimal surfaces. The edge arrays of finite tiles, that can be systematically enumerated using Delaney-Dress tiling theory, generate many - though not all - of the commonly encountered crystalline networks.

A variety of hyperbolic tilings can be formed using infinite tiles, whose edges are lines or trees. Projections of those examples generate more complex patterns. Packed trees result in multiple interpenetrating networks, which define “polycontinuous” space partitions. Packings of hyperbolic lines project to arrays of generalized helices, whose simplest examples are well-known rod packings. Generic examples are complex 3D weavings, whose knottedness can be captured by analyzing the homotopy of their quotient graphs (links) embedded in the relevant hyperbolic manifolds (the “minimal embeddings” of the links).

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